Electroweak Effects in Parton Distribution Functions

James Stirling
IPPP, Durham University

ESF Workshop, 12 September 2003

Motivation:

$$\left(\frac{\alpha_S}{2\pi}\right)^2 \simeq 0.004, \qquad \frac{\alpha}{2\pi} \simeq 0.001$$

... therefore electroweak effects could, in principle, be as important as NNLO QCD effects

(with A.D. Martin, R.G. Roberts and R.S. Thorne)

QCD-improved parton model

- DIS structure functions (e.g. F_2^{ep})

$$\mathcal{F}_i(x,Q^2) = \int_x^1 \frac{dy}{y}\Big\{\sum_j C_{ij}(y,\alpha_S)\, q_j(\frac{x}{y},Q^2) + C_{ig}(y,\alpha_S)\, g(\frac{x}{y},Q^2)\Big\}$$

- hadronic cross sections (e.g. $\sigma(p\bar{p}\to W+...)$)

$$d\sigma_X = \sum_{\text{partons } a,b} \int dx_a dx_b\, f_a(x_a,Q^2)\, f_b(x_b,Q^2)\, d\hat{\sigma}_{ab\to X}$$

- DGLAP evolution

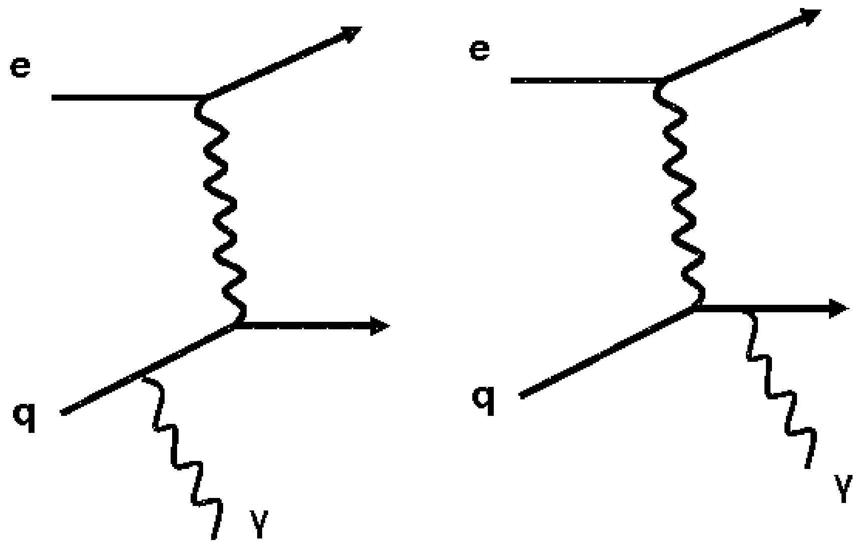
$$\frac{\partial q_i(x,\mu^2)}{\partial \log\mu^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{q_iq_j}(y,\alpha_S)\, q_j(\frac{x}{y},\mu^2) + P_{q_ig}(y,\alpha_S)\, g(\frac{x}{y},\mu^2)\Big\}$$

$$\frac{\partial g(x,\mu^2)}{\partial \log\mu^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{gq_j}(y,\alpha_S)\, q_j(\frac{x}{y},\mu^2) + P_{gg}(y,\alpha_S)\, g(\frac{x}{y},\mu^2)\Big\}$$

- NLO/NNLO: coefficient functions C, $\hat{\sigma}$ and splitting functions P evaluated to next-to-leading order/next-to-next-to-leading order in α_S in a particular factorisation/renormalisation scheme

QED effects

QED corrections to DIS include



$\Rightarrow$ mass singularity when $\gamma \parallel q$

$$\frac{\alpha}{2\pi} \langle e_q^2 \rangle \; \ln \left(\frac{Q^2}{m_q^2} \right) \simeq 0.01$$

for $Q = 100$ GeV, $m_q = 10$ MeV, $\langle e_q^2 \rangle = 5/18$.

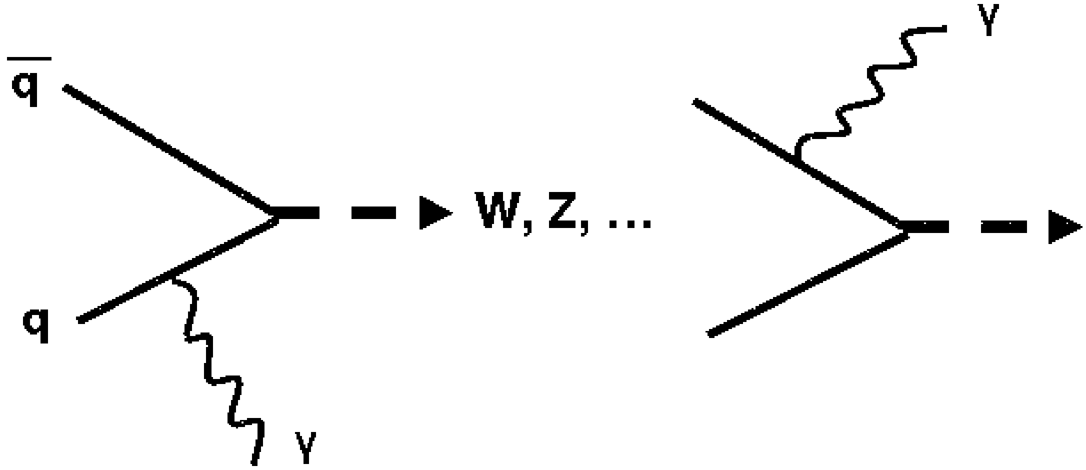
- such corrections included in standard QED radiative correction packages:

  `HERACLES`: Spiesberger et al., Comp. Phys. Comm. 69, 155 (1992)
  `HECTOR`: Arbuzov et al., hep-ph/9511434

  Note: interference of leptonic and partonic radiative corrections finite as $m_q \to 0$

- Issue: exactly what EW corrections have been applied in extraction of structure function measurements in DIS experiments?

- above QED collinear singularities are *universal* and can be absorbed into pdfs, exactly as for QCD collinear singularities, leaving finite (as $m_q \to 0$) $\mathcal{O}(\alpha)$ QED corrections in coefficient functions



- relevant for existing electroweak correction calculations for processes at Tevatron, LHC, e.g. W, Z, WH, ...

— see talks by Wackeroth and Krämer at this meeting, and U. Baur, S. Keller and D. Wackeroth, Phys. Rev. D59, 013002 (1999) for a full discussion of the formalism

QED–improved DGLAP equations

$$\frac{\partial q_i(x,\mu^2)}{\partial \log \mu^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{qq}(y)\ q_i(\frac{x}{y},\mu^2) + P_{qg}(y,\alpha_S)\ g(\frac{x}{y},\mu^2)\right\}$$

$$+ \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\left\{\tilde{P}_{qq}(y)\ e_i^2 q_i(\frac{x}{y},\mu^2) + P_{q\gamma}(y)\ e_i^2 \gamma(\frac{x}{y},\mu^2)\right\}$$

$$\frac{\partial g(x,\mu^2)}{\partial \log \mu^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{gq}(y)\ \sum_j q_j(\frac{x}{y},\mu^2)\right.$$

$$\left. + P_{gg}(y)\ g(\frac{x}{y},\mu^2)\right\}$$

$$\frac{\partial \gamma(x,\mu^2)}{\partial \log \mu^2} = \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{\gamma q}(y)\ \sum_j e_j^2\ q_j(\frac{x}{y},\mu^2)\right.$$

$$\left. + P_{\gamma\gamma}(y)\ \gamma(\frac{x}{y},\mu^2)\right\}$$

at leading order in α_S and α, where

$$\tilde{P}_{qq} = C_F^{-1} P_{qq}, \qquad P_{\gamma q} = C_F^{-1} P_{gq},$$

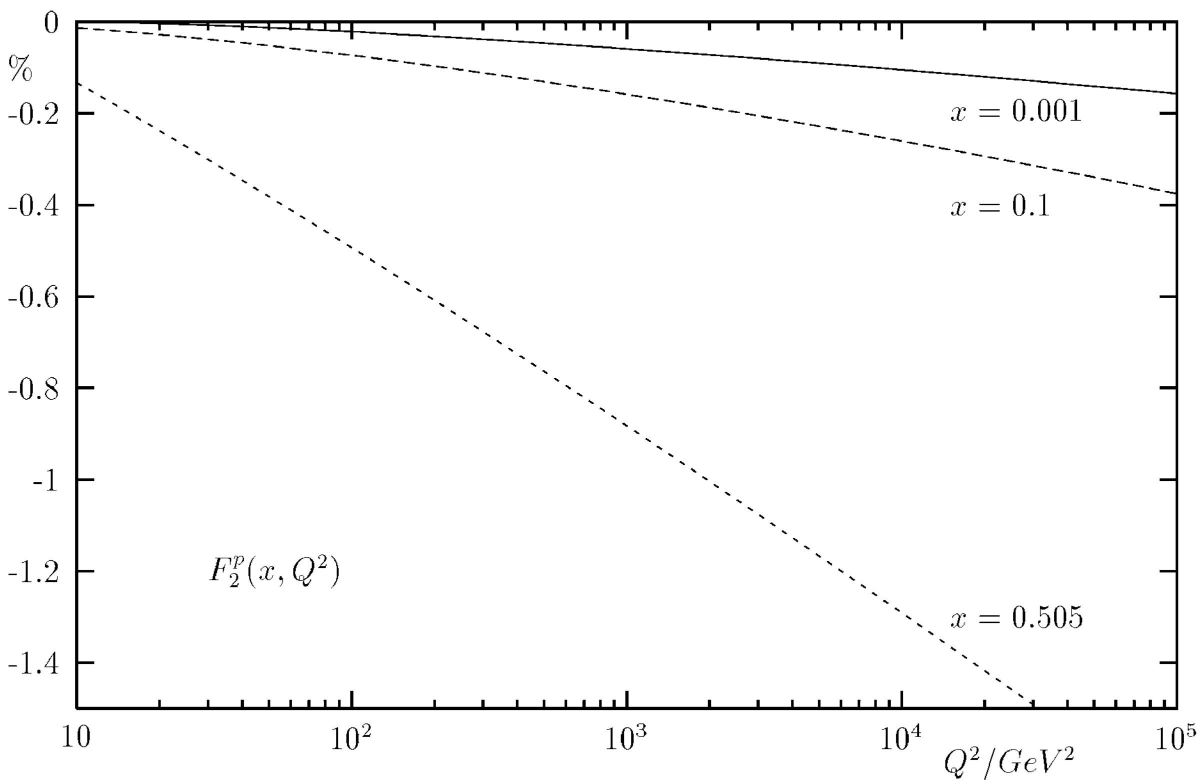
$$P_{q\gamma} = T_R^{-1} P_{qg}, \qquad P_{\gamma\gamma} = -\frac{2}{3}\sum_i e_i^2\ \delta(1-x)$$

and momentum is conserved:

$$\int_0^1 dx\ x\left\{\sum_i q_i(x,\mu^2) + g(x,\mu^2) + \gamma(x,\mu^2)\right\} = 1$$

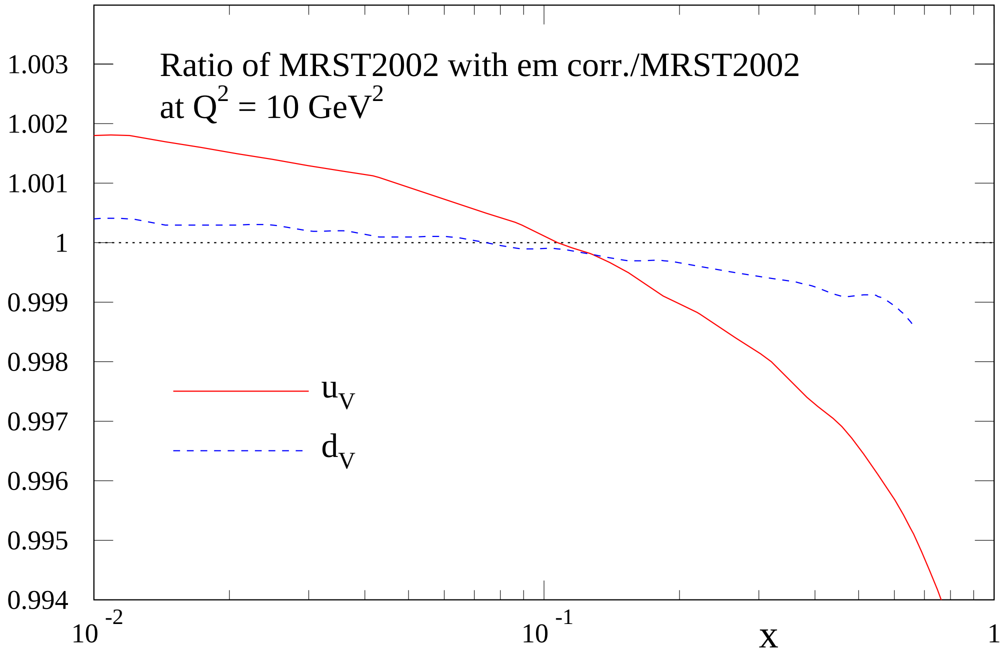
Note: in principle could introduce *different* factorisation scales for QCD, QED subtraction, thus $q(x, \mu^2_{F\mathrm{qcd}}, \mu^2_{F\mathrm{qed}})$ etc with DGLAP equations for evolution with respect to each scale

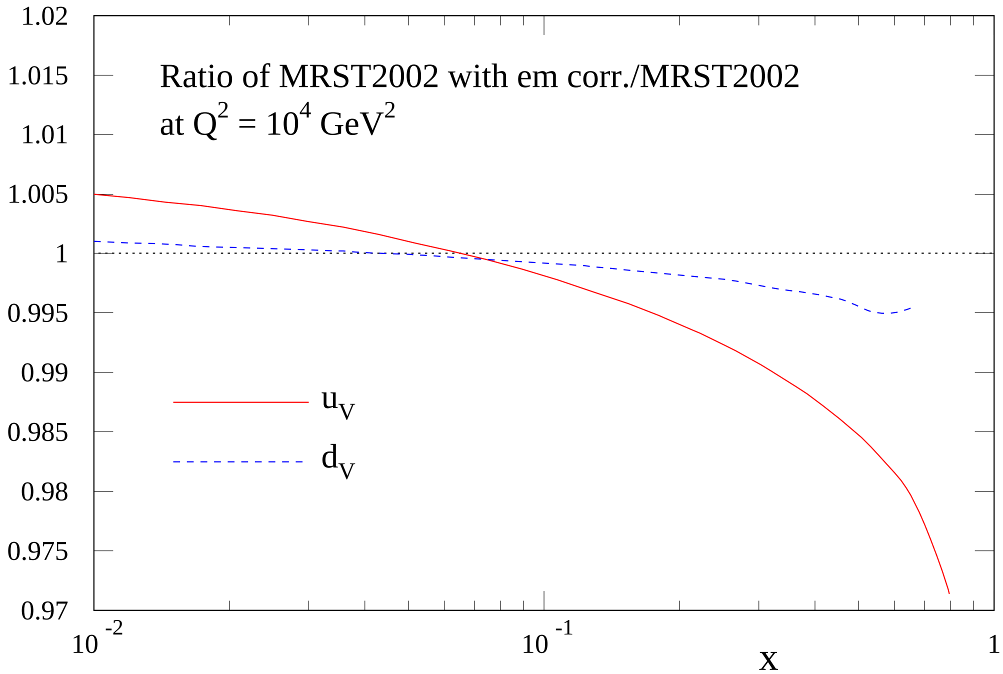
- first quantitative estimate of effect on pdfs by Spiesberger, Phys. Rev. D52, 4936 (1995)



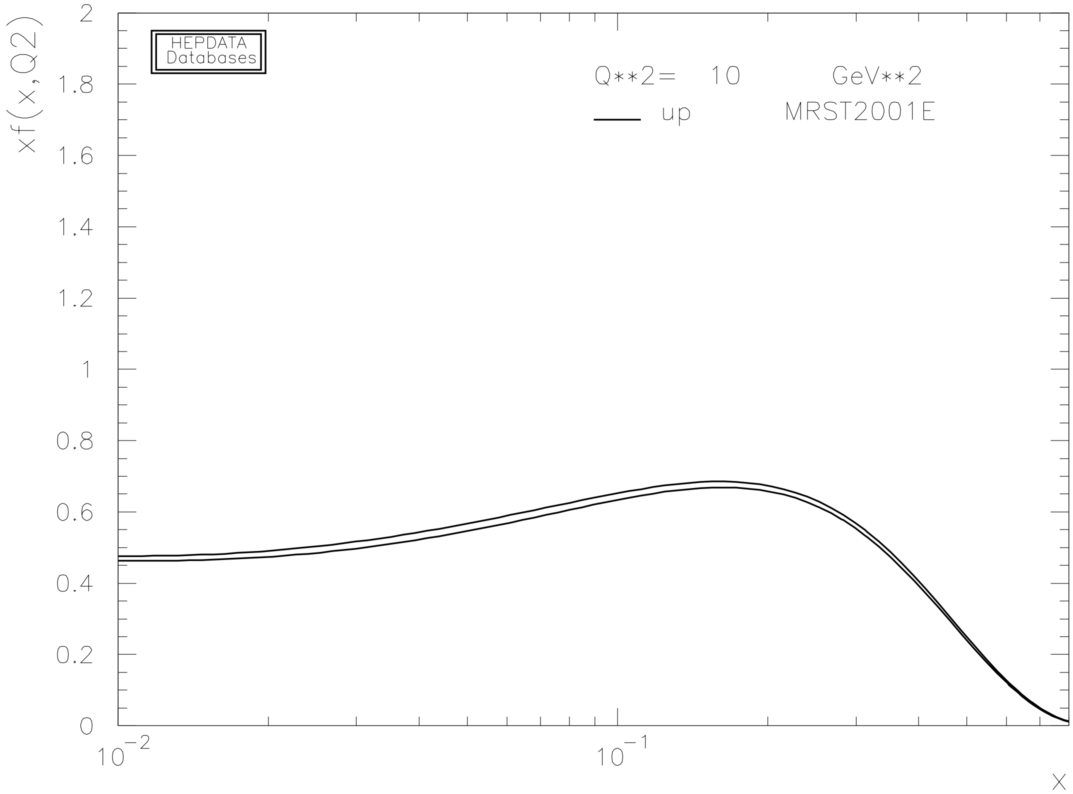
- new study by MRST in progress

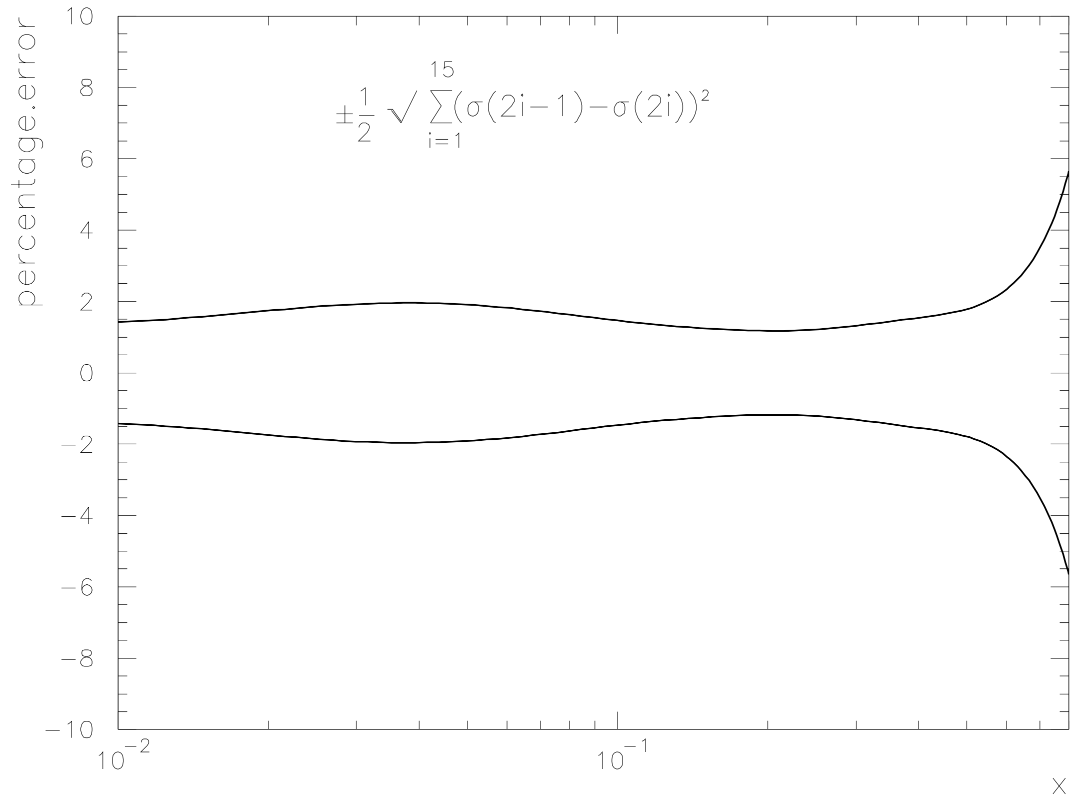
Effect of including em corrections to valence quark evolution



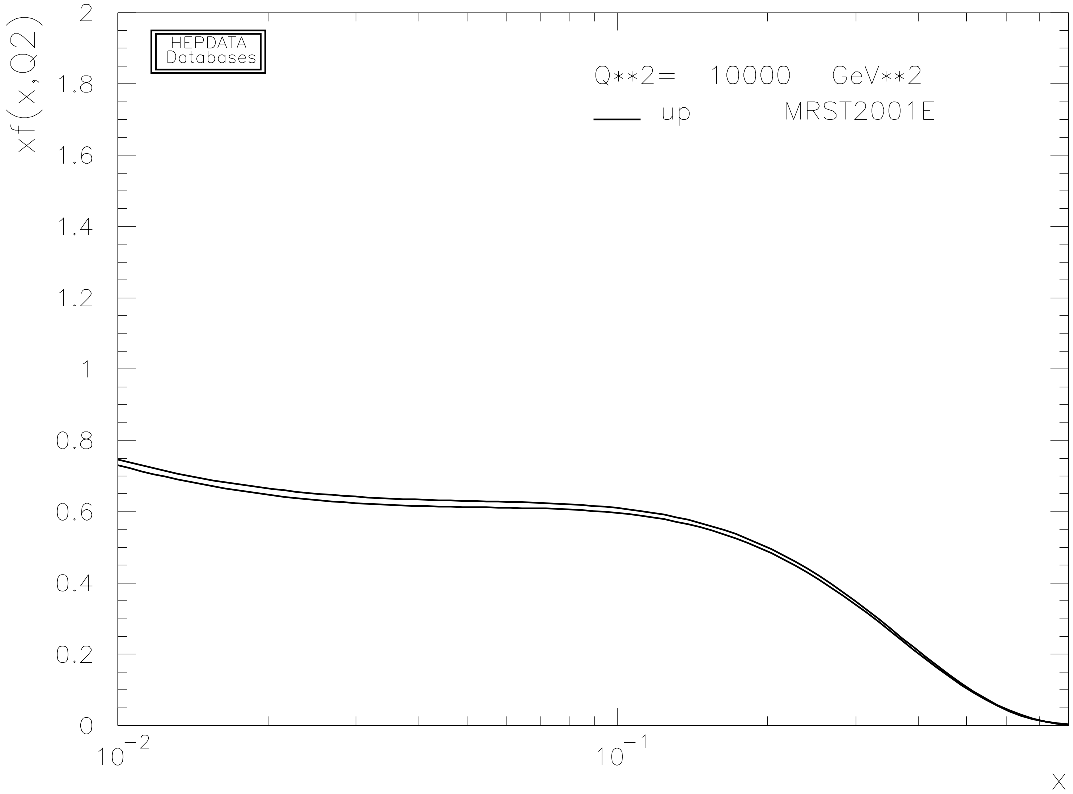


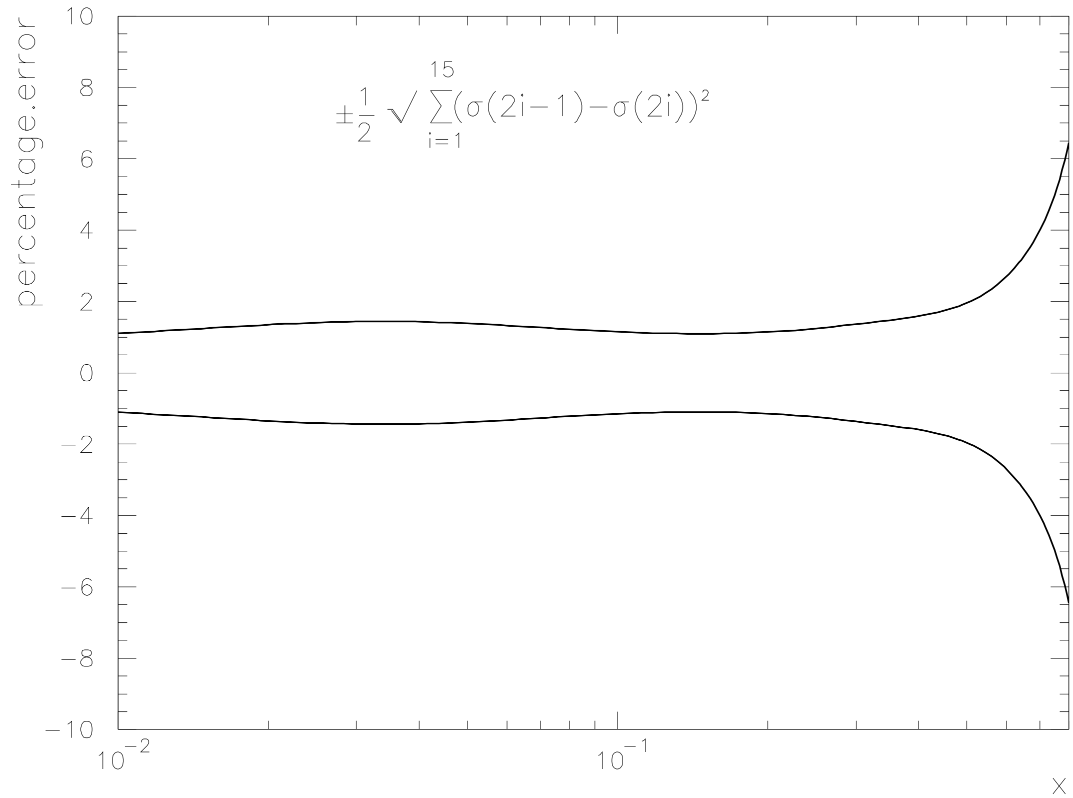
error on up distribution at $10\ \mathrm{GeV}^2$





error on up distribution at 10^4 GeV^2





comments

- effect on quark distributions is entirely negligible at small x where gluon contribution dominates DGLAP evolution

- at large x, effect only becomes noticeable (percent order) at very large Q^2, where it is equivalent to a slight shift in α_S:

$$\Delta\alpha_S(M_Z^2) \simeq +0.0003$$

cf. world average (global pdf fit) error of

$$\alpha_S^{\mathrm{NLO}}(M_Z^2) = 0.1165 \pm 0.002\ (\text{expt.})\ \pm 0.003\ (\text{theory})$$

(MRST, hep-ph/0308087)

- dynamic generation of photon parton distribution $\gamma(x, Q^2)$

conclusions

- the formalism exists for incorporating electroweak corrections into pdf analyses, i.e. global fits and DGLAP evolution

- photon radiation off quarks gives rise to $\gamma(x, Q^2)$
  Question: $\gamma(x, Q_0^2) = ??$

- quantitative effect on quark and gluon distributions is largest at large x and Q^2, but smaller than *current* pdf uncertainties

- need to ensure consistency between what EW corrections are/are not included in experimental measurements, e.g. of $F_2(x, Q^2)$, used in global fit analyses