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## **Combining the Top Quark Mass Results for Run 1 from CDF and D0**

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# Combining the Top Quark Mass Results for Run 1 from CDF and DØ

The Top Averaging Group<sup>1</sup>

For the CDF and DØ Collaborations

## 1 Introduction

CDF and DØ have published or submitted for publication eleven papers [1-11] that include direct experimental measurements of the top quark mass. Each experiment has found that the lepton+jets channel [1,5,6,9,10,11,12] gives the most precise result. However, CDF and DØ have also measured the top quark mass in other decay topologies. Both DØ [2,6] and CDF [3,4] have published additional results in the dilepton channels, while CDF [8] has published a result in the all-hadronic channel. Our aim in this note is to reduce the overall uncertainty on the top quark mass measurement by combining the most recent and final of these measurements [1,2,3,7,8].

This note combines the mass from the five separate measurements taking into account the statistical and systematic uncertainties as well as the correlations in the systematic uncertainties. The final Tevatron top mass based on Run 1 data is  $m_t = 174.3 \pm 3.2(\text{stat}) \pm 4.0(\text{syst}) \text{ GeV}/c^2$ . Combining the statistical and systematic uncertainties, the top mass is  $m_t = 174.3 \pm 5.1 \text{ GeV}/c^2$ .

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Decay Channel	CDF ( $\text{GeV}/c^2$ )	$D\bar{O}$ ( $\text{GeV}/c^2$ )
Lepton+jets	$175.9 \pm 4.8(\text{stat}) \pm 5.3(\text{syst})$	$173.3 \pm 5.6(\text{stat}) \pm 5.5(\text{syst})$
Dilepton	$167.4 \pm 10.3(\text{stat}) \pm 4.8(\text{syst})$	$168.4 \pm 12.3(\text{stat}) \pm 3.6(\text{syst})$
All-Hadronic	$186.0 \pm 10.0(\text{stat}) \pm 5.7(\text{syst})$	—

Table 1: *A summary of the published results on top mass for both the CDF and  $D\bar{O}$  experiment.*

## 2 Methodology

The results for the top quark mass for each of the decay channels published by CDF and  $D\bar{O}$  are given in Table 1. Each measurement has an associated statistical and systematic uncertainty. When combining these measurements, the statistical uncertainties can be treated as completely uncorrelated since the data samples are statistically independent. However, the systematic uncertainties are correlated and these correlations must be taken into account.

The systematic uncertainties in each of the measurements are assigned to one of six independent categories. These categories are:

1. Jet Energy Scale
2. Model for Signal
3. Monte Carlo Generator
4. Uranium Noise and Multiple Interactions
5. Model for Background
6. Method for Mass Fitting

The systematics for jet energy scale include information on the absolute jet energy corrections, calorimeter stability, underlying event, and relative jet energy corrections. The systematics for the signal model include initial and final state radiation effects, b-tagging bias, dependence upon parton distribution functions as well as variations in  $\Lambda_{QCD}$ . The systematic uncertainty on the Monte Carlo generator provides an estimate of sensitivity to the Monte Carlo generators by comparing HERWIG to PYTHIA or HERWIG to ISAJET. The fifth category, the background model, includes estimates of the effect of setting  $Q^2 = \langle p_t \rangle^2$  instead of  $Q^2 = M_W^2$  in VECBOS simulations of W+jets, the use of ISAJET fragmentation instead of HERWIG fragmentation as well as the effect of varying the background fraction attributed to

Systematic Category	CDF			DØ	
	Dilepton	Lep+jets	All-Had	Dilepton	Lep+jets
Jet Energy Scale	3.8	4.4	5.0	2.4	4.0
Model for Signal	2.8	2.6	1.8	1.7	1.9
MC Generator	0.6	0.1	0.8	0.0	0.0
Uranium Noise/Mult Int	0.0	0.0	0.0	1.3	1.3
Model for Background	0.3	1.3	1.7	1.0	2.5
Method for Mass Fitting	0.7	0.0	0.6	1.1	1.5

Table 2: *Systematic uncertainties for the five published top mass values from CDF and DØ in GeV/c<sup>2</sup>. The different categories are described in the text.*

QCD. Finally the systematic uncertainty in mass fitting takes account of the finite sizes of the Monte Carlo samples, impact of jet permutations, and other fitting biases.

For each mass analysis, the systematic uncertainties assigned to a given category are summed in quadrature. The results are shown in Table 2. In the CDF lepton+jets analysis, the systematic uncertainty due to finite Monte Carlo statistics is included in the statistical uncertainty. Furthermore, CDF does not have uranium noise and includes the systematic uncertainty due to multiple interactions in the first category. Likewise, in the DØ analyses, the systematic uncertainty associated with the comparison of HERWIG to ISAJET is included in the signal model uncertainty.

For each of the six categories, the systematic uncertainties in the measurements were assumed to be either uncorrelated or completely correlated. For instance, the uncertainty on the jet energy scale is taken to be 100% correlated within an experiment since all of those analyses use the same detector as well as the same jet clustering algorithm. On the other hand, the jet energy scale is not correlated between experiments. The uncertainties in modeling of signal via the Monte Carlo generator are assumed to be 100% correlated within each experiment as well as between experiments, since all of the analyses use the HERWIG Monte Carlo generator to simulate  $t\bar{t}$  events. Finally, for a given  $t\bar{t}$  decay channel, the uncertainties on the background model are 100% correlated between experiments. However, these uncertainties are not correlated between channels since the background processes are different. The correlation coefficients between the ten pairs of analyses are given in Table 3.

We combine the five mass measurements using standard methods described in Appendix A of this document, taking into account the statistical uncertainties, systematic uncertainties, and correlations discussed above. The result is a Tevatron top mass value of

$$m_t = 174.3 \pm 5.1 \text{ GeV}/c^2,$$

or, separating the systematic and statistical uncertainties:

Category	ab	ac	bc	ad	ae	bd	be	cd	ce	de
Jet Energy Scale	1	1	1	0	0	0	0	0	0	1
Model for Signal	1	1	1	1	1	1	1	1	1	1
MC Generator	1	1	1	1	1	1	1	1	1	1
Uranium Noise/Mult Int	0	0	0	0	0	0	0	0	0	1
Model for Background	0	0	0	1	0	0	0	0	1	0
Method for Mass Fitting	0	0	0	0	0	0	0	0	0	0

Table 3: *Correlation coefficients used in determining the combined uncertainty. A “1” indicates 100% correlation and a “0” indicates no correlation. The key for the horizontal axis is a = CDF Lepton+jets, b = CDF All Hadronic, c = CDF Dilepton, d = DØ Lepton+jets, e = DØ Dilepton channel.*

$$m_t = 174.3 \pm 3.2(\text{stat}) \pm 4.0(\text{syst}) \text{ GeV}/c^2.$$

In the calculation, the central value can be written as the weighted sum of five central input values. These weights, which depend upon the statistical and systematic uncertainties as well as the correlations, are listed in Table 4. We define a statistical uncertainty for the combined result as the quadratic sum of the weighted individual statistical uncertainties. That combined statistical uncertainty is 3.2 GeV/c<sup>2</sup>. The combined systematic uncertainty, defined as the difference in quadrature between the total uncertainty and the statistical uncertainty is then 4.0 GeV/c<sup>2</sup>.

In deciding which systematics are correlated and which are not, we have “erred” on the side of caution. Clearly, sometimes errors are correlated, but not necessarily at the 100% level. We have not tried to determine the “proper” amount of correlation. If we were to assume that all systematic uncertainties are 100% correlated, we get a mass of 174.0 ± 5.9 GeV/c<sup>2</sup>. Similarly, if we were to treat each systematic uncertainty as completely uncorrelated, we find a combined mass of 174.6 ± 4.2 GeV/c<sup>2</sup>. Our result of 174.3 ± 5.1 GeV/c<sup>2</sup> is midway between these two extremes.

### 3 Summary

We have combined the five published top mass measurements from CDF and DØ, taking into account the statistical and correlated systematic uncertainties. The Tevatron average based on these Run 1 results is  $m_t = 174.3 \pm 3.2(\text{stat}) \pm 4.0(\text{syst}) \text{ GeV}/c^2$ .

Measurement	Relative Weight
CDF Lepton+Jets	0.35
CDF Dilepton	0.10
CDF All-Hadronic	0.10
DØ Lepton+jets	0.34
DØ Dilepton	0.11

Table 4: *Relative weights of the five individual mass measurements in the combined result. (One should be careful not to equate relative weights with relative importance of the measurements)*

## References

- [1] T. Affolder *et al.*, *Measurement of the Top Quark Mass Using the Collider Detector at Fermilab*, submitted for publication in Phys. Rev. D.
- [2] B. Abbott *et al.*, *Measurement of the Top Quark Mass in the Dilepton Channel*, accepted for publication by Phys Rev. D, hep-ex/9808029.
- [3] F. Abe *et al.*, *Measurement of the Top Quark Mass with the Collider Detector at Fermilab*, Phys. Rev. Lett. **82**, 271 (1999).
- [4] F. Abe *et al.*, *Measurement of the Top Quark Mass and  $t\bar{t}$  Production Cross Section from Dilepton Events at the Collider Detector at Fermilab*, Phys. Rev. Lett. **80**, 2779 (1998).
- [5] F. Abe *et al.*, *Measurement of the Top Quark Mass*, Phys. Rev. Lett. **80**, 2767 (1998).
- [6] B. Abbott *et al.*, *Direct Measurement of the Top Quark Mass using Dilepton Events*, Phys. Rev. Lett. **80**, 2063 (1998).
- [7] B. Abbott *et al.*, *Direct Measurement of the Top Quark Mass at DØ*, Phys. Rev. D **58**, 052001 (1998).
- [8] F. Abe *et al.*, *First Observation of the All-Hadronic decay of  $t\bar{t}$  pairs*, Phys. Rev. Lett. **79**, 1992 (1997).
- [9] S. Abachi *et al.*, *Direct Measurement of the Top Quark Mass*, Phys. Rev. Lett. **79**, 1197 (1997).
- [10] S. Abachi *et al.*, *Observation of the Top Quark*, Phys. Rev. Lett. **74**, 2632 (1995).
- [11] F. Abe *et al.*, *Observation of Top Quark Production in  $p\bar{p}$  collisions at  $\sqrt{s} = 1.8$  TeV*, Phys. Rev. Lett. **74**, 2626 (1995).

- [12] F. Abe *et al.*, *Evidence for Top Quark Production in  $p\bar{p}$  collisions at  $\sqrt{s} = 1.8$  TeV*, Phys. Rev. Lett. **73**, 225 (1994) and Phys. Rev. D**50**, 2966 (1994).



# A Appendix A: Procedure for combining results from two experiments

## A.1 Notation

We consider two experiments  $a$  and  $b$ , each of which measures the same quantity  $Q$ . The best fit values are  $Q^\alpha$ , the statistical uncertainties are  $T^\alpha$ , and the systematic uncertainties are  $y_i^\alpha$ , where  $\alpha$  labels the experiments and  $i$  runs over the same  $N$  major categories of systematic uncertainties for each experiment. Each major category can contain several uncertainty components, and these components can also be correlated. However, the categories are chosen so as to be uncorrelated within either experiment. Consequently, for either experiment, the total systematic uncertainty is just the quadrature sum of the major-category systematic uncertainties.

Hence for either experiment we can write the total systematic uncertainty as:

$$Y^\alpha = \sqrt{\sum_{i=1}^N (y_i^\alpha)^2} \quad (1)$$

and the total uncertainty as

$$S^\alpha = \sqrt{(Y^\alpha)^2 + (T^\alpha)^2}. \quad (2)$$

## A.2 Correlation among components of uncertainty in each experiment

To combine the two experiments, we define a correlation coefficient  $\rho_i$  for each major category  $i$  of systematic uncertainty. If the uncertainties are not correlated, we set  $\rho_i = 0$ ; if they are fully correlated between experiments, we set  $\rho_i = 1$ . These are the most common choices for the  $\rho_i$ . However, nothing in the following presupposes any particular set of values for the  $\rho_i$ .

The coefficient of mutual correlation  $\rho_i$  for the systematic uncertainty in major category  $i$  is

$$\rho_i = \frac{\langle \delta_i^a \delta_i^b \rangle}{\sqrt{\langle (\delta_i^a)^2 \rangle \langle (\delta_i^b)^2 \rangle}}, \quad (3)$$

where  $\delta_i^\alpha$  is the deviation in  $Q^\alpha$  associated with systematic uncertainty  $i$ , and the averages are taken over an ensemble of possibilities that reflect different (and possibly mutually correlated) choices of analysis methods.

### A.3 Correlation between total errors in each experiment

Next, we evaluate the coefficient of correlation  $\rho$  for the total deviations  $\Delta^\alpha$  from the true value of  $Q$ . These total deviations have contributions both from the systematic deviations  $\delta_i^\alpha$  and from the statistical deviations  $\xi^\alpha$  in both experiments:

$$\Delta^\alpha = \sum_{i=1}^N \delta_i^\alpha + \xi^\alpha . \quad (4)$$

The individual systematic, statistical, and total uncertainties for either experiment are related to these deviations as usual:

$$y_i^\alpha = \sqrt{\langle (\delta_i^\alpha)^2 \rangle}, \quad T^\alpha = \sqrt{\langle (\xi^\alpha)^2 \rangle}, \quad S^\alpha = \sqrt{\langle (\Delta^\alpha)^2 \rangle} . \quad (5)$$

The correlation coefficient  $\rho$  for the total deviations between the two experiments is therefore:

$$\rho \equiv \frac{\langle \Delta^a \Delta^b \rangle}{\sqrt{\langle (\Delta^a)^2 \rangle \langle (\Delta^b)^2 \rangle}} .$$

Substituting  $(S^\alpha)^2$  for  $\langle (\Delta^\alpha)^2 \rangle$  in the denominator, and expressing  $\langle \Delta^a \Delta^b \rangle$  in the numerator in terms of the  $\delta_i^\alpha$  and the  $\xi^\alpha$ , we exploit the lack of correlation between the  $\xi^\alpha$  and other terms, and between  $\delta_i^\alpha$  and  $\delta_j^\alpha$  for  $i \neq j$ , to write the total correlation coefficient as:

$$\rho = \frac{\sum_{i=1}^N \langle \delta_i^a \delta_i^b \rangle}{S^a S^b} . \quad (6)$$

The numerator can be simplified further. By definition,

$$\rho_i \equiv \frac{\langle \delta_i^a \delta_i^b \rangle}{y_i^a y_i^b} . \quad (7)$$

With this substitution, the correlation coefficient  $\rho$  between the total errors  $S^a$  and  $S^b$  is finally:

$$\rho = \frac{\sum_{i=1}^N \rho_i y_i^a y_i^b}{S^a S^b} . \quad (8)$$

## A.4 Covariance matrix

In the  $\{a, b\}$  basis, in terms of  $\rho$ , the covariance matrix is

$$\mathcal{S} = \begin{pmatrix} (S^a)^2 & \rho S^a S^b \\ \rho S^a S^b & (S^b)^2 \end{pmatrix}. \quad (9)$$

The inverse of this matrix is used to define a  $\chi^2$  for the hypothesis that each experiment measures the same quantity  $Q$ :

$$\chi^2 = \mathcal{Q}^t \mathcal{S}^{-1} \mathcal{Q}, \quad (10)$$

where

$$\mathcal{Q} = \begin{pmatrix} \langle Q \rangle - Q^a \\ \langle Q \rangle - Q^b \end{pmatrix} \quad (11)$$

and  $\langle Q \rangle$  is the combined best-fit estimate of  $Q$ .

## A.5 Combining experimental results

Minimizing  $\chi^2$  with respect to  $\langle Q \rangle$ , the best fit quantity obtained by combining the two experiments is:

$$\langle Q \rangle = w^a Q^a + w^b Q^b, \quad (12)$$

where:

$$w^a \equiv \frac{S^b(S^b - \rho S^a)}{(S^a)^2 + (S^b)^2 - 2\rho S^a S^b} \quad \text{and} \quad w^b \equiv \frac{S^a(S^a - \rho S^b)}{(S^a)^2 + (S^b)^2 - 2\rho S^a S^b}. \quad (12')$$

Assuming Gaussian errors, the curvature of  $\chi^2$  vs.  $\langle Q \rangle$  yields the total uncertainty on  $\langle Q \rangle$ , which can be written as:

$$S_{\langle Q \rangle} = \sqrt{\frac{(S^a S^b)^2 (1 - \rho^2)}{(S^a)^2 + (S^b)^2 - 2\rho S^a S^b}}. \quad (13)$$

There are several ways of decomposing this total uncertainty into statistical and systematic components. Here, we define the combined statistical uncertainty on  $\langle Q \rangle$  as:

$$T_{\langle Q \rangle} = \sqrt{(w^a T^a)^2 + (w^b T^b)^2}. \quad (14)$$

The combined systematic uncertainty on  $\langle Q \rangle$  is then:

$$Y_{\langle Q \rangle} = \sqrt{S_{\langle Q \rangle}^2 - T_{\langle Q \rangle}^2}. \quad (15)$$

## A.6 Equivalence of Current Technique and “Correlated Error” Method

The “correlated error” method used in the past to combine Tevatron mass measurements (top mass in  $\ell$ +jets and the  $W$  mass) is equivalent to the procedure described above, provided that proper definitions of “correlated” and “statistical” uncertainties are used in the process. The “correlated error”  $\sigma_c$  is defined by

$$\sigma_c^2 = \rho S^a S^b , \quad (16)$$

and the “statistical error”  $\sigma_t^\alpha$  of either experiment is defined by

$$(\sigma_t^\alpha)^2 = (S^\alpha)^2 - \sigma_c^2 . \quad (17)$$

The same combined result  $\langle Q \rangle$  is then also given by the familiar relation

$$\langle Q \rangle = \frac{Q^a/(\sigma_t^a)^2 + Q^b/(\sigma_t^b)^2}{1/(\sigma_t^a)^2 + 1/(\sigma_t^b)^2} , \quad (18)$$

and the same total uncertainty  $S_{\langle Q \rangle}$  is given by

$$S_{\langle Q \rangle}^2 = \frac{1}{1/(\sigma_t^a)^2 + 1/(\sigma_t^b)^2} + \sigma_c^2 . \quad (19)$$

## A.7 Summary

The result of combining two measurements  $Q^a$  and  $Q^b$  is

$$\langle Q \rangle \pm T_{\langle Q \rangle}(\text{stat}) \pm Y_{\langle Q \rangle}(\text{syst}) = \langle Q \rangle \pm S_{\langle Q \rangle}(\text{tot}) , \quad (20)$$

with  $\langle Q \rangle$ ,  $T_{\langle Q \rangle}$ ,  $Y_{\langle Q \rangle}$ , and  $S_{\langle Q \rangle}$  given by Eqs. (12) through (15).

## B Appendix B: Generalization to combining more than 2 experimental results

The formalism developed in the main part of this note is readily extended to the case of combining more than two experimental results. Consider  $R$  such results, with  $1 \leq \alpha \leq R$ . In Eq. (3) the coefficients of mutual correlation for each specific systematic category of uncertainties must now be indexed by  $a$  and  $b$ :

$$\rho_i^{ab} = \frac{\langle \delta_i^a \delta_i^b \rangle}{\sqrt{\langle (\delta_i^a)^2 \rangle \langle (\delta_i^b)^2 \rangle}} . \quad (3a)$$

The correlation coefficients between total uncertainties for each pair of experiments form the elements of a correlation matrix:

$$\rho^{ab} = \frac{\sum_{i=1}^N \rho_i^{ab} y_i^a y_i^b}{S^a S^b} . \quad (8a)$$

The elements of the  $R \times R$  covariance matrix  $S$  become

$$S^{ab} = \rho^{ab} S^a S^b . \quad (9a)$$

Combining the experimental results proceeds by minimizing a similar  $\chi^2$ , but the results need to be expressed more generally. Defining  $\mathcal{H}^{ab}$  as an element of the inverse of  $S$ , the result of combining the measurements is:

$$\langle Q \rangle = \sum_{a=1}^R w^a Q^a , \quad (12a)$$

where:

$$w^a \equiv \frac{\sum_{b=1}^R \mathcal{H}^{ab}}{\sum_{a,b=1}^R \mathcal{H}^{ab}} , \quad (12'a)$$

and the total error becomes

$$S_{\langle Q \rangle} = \frac{1}{\sqrt{\sum_{a,b=1}^R \mathcal{H}^{ab}}} . \quad (13a)$$

The combined statistical uncertainty generalizes to:

$$T_{\langle Q \rangle} = \sqrt{\sum_{\alpha=1}^R (w^\alpha T^\alpha)^2} , \quad (14a)$$

while the combined systematic error is still obtained from Eq. (15).