Fermilab FERMILAB-FN-0716 August 2002

CDF note 5888
DØ note 3963
FERMILAB-FN-0716

Combination of CDF and DØ Results on W Boson Mass and Width

Tevatron Electroweak Working Group*
and the CDF and DØ Collaborations

July 22, 2002

Abstract

The results based on 1992-95 data (Run 1) from the CDF and DØ experiments on the measurements of the W boson mass and width are presented, along with the combined results. We report a Tevatron collider average $M_W = 80.456 \pm 0.059$ GeV and a preliminary world average $M_W = 80.451 \pm 0.032$ GeV. We also report the Tevatron collider average of the directly measured W boson width $\Gamma_W = 2.115 \pm 0.105$ GeV, and the preliminary world average $\Gamma_W = 2.135 \pm 0.069$ GeV. We describe the calculation of the covariance matrix between the direct W mass and width measurements. Assuming the validity of the standard model, we combine the directly measured W boson width with the width extracted from the ratio of W and Z boson leptonic partial cross sections. This combined result for the Tevatron is $\Gamma_W = 2.160 \pm 0.047$ GeV and the preliminary world average is $\Gamma_W = 2.158 \pm 0.042$ GeV. We also use the measurements of the direct total W width and the leptonic branching ratio to extract the leptonic partial width $\Gamma(W \to e\nu) = 220.6 \pm 12.2$ MeV.

1 Introduction

We document the procedure used to combine the results from 1992-95 (Run 1) CDF and DØ data on the measurements of the W boson mass and

*contact person: Ashutosh Kotwal (kotwal@phy.duke.edu)

width. We summarize the results and the various sources of uncertainty, and identify those sources that produce correlated uncertainty between the two experiments' results. Based on publicly available data, we present the combined results.

The directly measured W boson mass and width correspond to the pole mass M_W and pole width Γ_W in the Breit-Wigner line shape with energy-dependent width, as defined by

$$\frac{d\sigma}{dQ} = \mathcal{L}_{q\bar{q}}(Q) \frac{Q^2}{(Q^2 - M_W^2)^2 + Q^4 \Gamma_W^2 / M_W^2} \quad , \tag{1}$$

where Q is the center-of-mass energy of the annihilating partons. $\mathcal{L}_{q\bar{q}}(Q)$ represents the parton-luminosity skewing factor in hadron-hadron collisions

$$\mathcal{L}_{q\bar{q}}(Q) = \frac{2Q}{s} \sum_{i,j} \int_{Q^2/s}^{1} \frac{dx}{x} f_i(x, Q^2) f_j(Q^2/sx, Q^2) \quad , \tag{2}$$

where $f_{i,j}$ represent the respective parton distribution functions and s is the hadron-hadron center-of-mass energy.

The W decay channels used for these measurements are the electron + neutrino channel (by CDF and DØ) and the muon + neutrino channel (by CDF). The W boson mass and width are extracted by analysing the Jacobian edge and the high mass tail respectively of the transverse mass (m_T) distribution

$$m_T = \sqrt{2\, p_T(e)\, p_T(\nu)\, (1 - \cos(\phi(e) - \phi(\nu)))} \quad . \tag{3}$$

DØ has also measured the W boson mass by analysing the Jacobian edge in the electron and neutrino transverse momentum (p_T) distributions. The CDF result for the W boson mass is quoted using the m_T fit, while the DØ result combines the m_T fit and the lepton p_T fits taking the correlations into account.

The W boson width is also extracted from the measured ratio of partial cross sections

$$\begin{aligned} R &\equiv \frac{\sigma_W \cdot B(W \to e\nu)}{\sigma_Z \cdot B(Z \to ee)} \\ &= \frac{\sigma_W}{\sigma_Z} \frac{\Gamma_Z}{\Gamma(Z \to ee)} \frac{\Gamma(W \to e\nu)}{\Gamma_W} \end{aligned} \tag{4}$$

by using as inputs the calculated ratio of total cross sections, the measured $Z \to ee$ branching ratio from LEP and the standard model calculation of the partial width $\Gamma(W \to e\nu)$.

2 W Boson Mass

The Run 1 W boson mass measurements from CDF [1] and DØ [2] are

$$\begin{aligned} M_W &= 80.433 \pm 0.079 \text{ GeV (CDF)} \\ M_W &= 80.483 \pm 0.084 \text{ GeV (DØ)} \end{aligned} \tag{5}$$

We discuss the sources of uncertainty and classify them as being either uncorrelated between the two experimental results, or (partially or completely) correlated.

2.1 Uncorrelated Uncertainties

The measurement and analysis techniques used by both experiments rely extensively on internal calibration and collider data to measure detector response and constrain theoretical model inputs. The bulk of the uncertainty is therefore uncorrelated. We itemize the uncorrelated sources below. The following discussion also applies to the uncorrelated uncertainties in the direct measurement of the W boson width (see Section 3).

- W statistics in the kinematic distributions used for the mass fits.

- Detector energy response and resolution measured using resonances (Z, ψ, Υ and π^0). Model uncertainty from resonance line shapes is negligible. These data are used for the calibration of lepton energy response (calorimetry and tracking for electrons and tracking for muons). The Z data are also used for calibrating the calorimeter response to the hadronic activity recoiling against the vector boson. In the CDF analysis, the lepton response and resolution and the hadronic recoil model are constrained independently for the electron and muon channel. In the internal CDF combination of these measurements, uncertainties in the lepton and recoil models are taken to be uncorrelated between channels.

- Selection biases and backgrounds are unique to each experiment and are measured mostly from collider data, with some input from detector simulation for evaluating selection bias. The uncertainty in the calculated $W \to \tau\nu \to l\nu\bar{\nu}\nu$ background is negligible. These uncertainties are uncorrelated between the CDF electron and muon channel measurements.

- The distribution of the transverse momentum (p_T) of the W boson is a model input, which each experiment constrains individually by fitting the Z boson p_T distribution. Phenomenological models such as that of Ellis, Ross and Veseli or that of Ladinsky and Yuan [4] are treated as empirical functions which, after folding in the detector response, adequately describe the observed $p_T(Z)$ distribution. The p_T distribution is specified by model parameters along with Λ_{QCD} and the parton distribution functions (PDFs). The uncertainty is dominated by Z statistics, with small dependence on the PDFs and Λ_{QCD}. The latter introduces a small correlation between the two experiments which can be neglected at this level. A potentially correlated uncertainty in the theoretical relationship between the W boson and the Z boson p_T spectra is assumed to be negligible. There is a small (3 MeV) correlated component in the $p_T(W)$ uncertainty between the CDF electron and muon channel results.

- The backgrounds sources are $Z \to ll$ where one of the leptons is lost, $W \to \tau\nu \to l\nu\bar{\nu}\nu$, and misidentified QCD jet events. The $Z \to ll$ background is estimated using individual detector simulations. The uncertainty on the $W \to \tau\nu \to l\nu\bar{\nu}\nu$ background is negligible. The jet misidentification background is estimated by using loosely defined lepton data samples which enhances the background contribution (DØ), or by selecting lepton candidates that fail quality cuts (CDF). While the techniques are similar in principle they differ in detail. CDF has also confirmed the jet misidentification background estimate using a photon conversion sample. The background uncertainties and cross-checks are statistics-limited and therefore independent.

Table 1 shows the contributions to the uncertainty which are uncorrelated between the CDF and DØ measurements.

Table 1: Uncorrelated uncertainties (MeV) in the CDF and DØ W boson mass measurements. W boson decay channels used (e, μ) are listed separately.

Source	CDF μ	CDF e	DØ e
W statistics	100	65	60
Lepton scale	85	75	56
Lepton resolution	20	25	19
$p_T(W)$	20	15	15
Recoil model	35	37	35
Selection bias	18	-	12
Backgrounds	25	5	9

2.2 Correlated Uncertainties

Sources of correlated uncertainty are associated with the modelling of W production and decay, which we itemize below. The uncertainties are taken to be fully correlated between CDF and DØ, with possibly different magnitudes.

- The W boson kinematic distributions used in the fits are invariant under longitudinal boosts because they are derived from transverse quantities. The sensitivity to the PDFs arises because of acceptance cuts on the charged lepton rapidity. As the rapidity acceptance increases the sensitivity to PDFs reduces. The DØ W boson mass measurement includes electrons up to pseudorapidity $|\eta| < 2.5$, and the CDF measurement includes electrons and muons up to $|\eta| < 1.0$. The PDF uncertainty is correlated but different for the two measurements.

- The Breit-Wigner line shape is skewed by the mass-dependent parton luminosity. This is a small contribution which DØ quotes separately, but CDF subsumes into the overall PDF uncertainty.

- QED radiative corrections in leptonic W boson decays are evaluated by both experiments using the Berends and Kleiss [5] calculation. The uncertainty is evaluated by comparing to the PHOTOS [6] program and/or the calculation of Baur *et al.* [7]. The higher order QED effects have a different impact on the electron and muon channel measurements from CDF and the electron measurement from DØ due to differences in energy measurement techniques. We find that in the combined electron and muon channel result of CDF, the effective uncertainty due to QED radiative corrections is 11 MeV. We take this contribution to be fully correlated with the corresponding uncertainty in the DØ result.

- The W width input into the W boson mass measurement is provided for differently by CDF and DØ. CDF uses the standard model prediction for Γ_W for the fitted value of M_W and the resulting uncertainty is negligible. DØ uses the indirect measurement of the W width which is extracted from the DØ measurement of the ratio $\sigma(W \to e\nu)/\sigma(Z \to ee)$. For the purpose of combining the results, we take the 10 MeV uncertainty quoted by DØ to be the correlated error.

Table 2 shows the correlated systematic uncertainties, taken from [1] and [3] respectively.

Table 2: Systematic uncertainties (MeV) from correlated sources in the W boson mass measurements.

Source	CDF	DØ
PDF & parton luminosity	15	$7 \oplus 4$
Radiative Corrections	11	12
Γ_W	10	10

2.3 Combination of Results

We use the Best Linear Unbiased Estimate [8] method, which is also used in [3], to construct the covariance matrix between the CDF and DØ measurements. For each source of correlated error, we construct a 2-component vector $\delta_i \vec{M}_W$ whose components are the individual uncertainties quoted in Table 2, *i.e.* $\delta_i \vec{M}_W = (\delta_i M_W^{\mathrm{CDF}}, \delta_i M_W^{\mathrm{D}\emptyset})$ for the i^{th} source of uncertainty. The contribution to the covariance matrix from each source is given by $V_i = \delta_i \vec{M}_W (\delta_i \vec{M}_W)^T$, where T indicates the transpose. The various sources of error are assumed to be uncorrelated with each other, hence we add the individual covariance matrices V_i to obtain $V = \sum_i V_i$. This procedure gives us the off-diagonal term in the total covariance matrix V. The diagonal terms are obtained from the square of each measurement's total error. The square root of the off-diagonal covariance matrix element $\sqrt{V_{12}}$ gives the total correlated error between the CDF and DØ measurements of 19 MeV. The correlation coefficient, defined by $V_{12}/\sqrt{V_{11}V_{22}}$, is $19^2/(79 \times 84) = 0.054$.

The combined W mass M_W for the set of two W mass measurements m_i and their covariance matrix V is given by

$$M_W = (\sum_{i,j=1}^{2} H_{ij}\ m_j) / (\sum_{i,j=1}^{2} H_{ij}) \quad , \tag{6}$$

where $H \equiv V^{-1}$ and i, j run over the two W mass measurements being combined. The combined error is given by

$$\sigma(M_W) = (\sum_{i,j=1}^{2} H_{ij})^{-1/2} \quad , \tag{7}$$

and the χ^2 for the combination is given by

$$\chi^2 = \sum_{i,j=1}^{2} (m_i - M_W)\ H_{ij}\ (m_j - M_W) \quad . \tag{8}$$

Using this procedure, we obtain the combined result for the Tevatron collider

$$M_W = 80.456 \pm 0.059 \ \ \mathrm{GeV} \quad , \tag{9}$$

with $\chi^2 = 0.2$ and probability of 66%.

We note that the various W mass measurements from DØ are internally combined by DØ [3] using the same technique that we describe above. CDF combines its internal measurements [1] using a slightly different formulation, where the measurements are combined using only the uncorrelated errors, and then the correlated errors are added in quadrature. When the correlated errors are small with positive correlation coefficients, as we have here, the two formulations give very similar results.

The combination of the Tevatron collider average with the UA2 measurement [10] of

$$M_W^{\mathrm{UA2}} = 80.36 \pm 0.37 \ \ \mathrm{GeV} \tag{10}$$

with a common uncertainty of 25 MeV yields

$$M_W^{p\bar{p}} = 80.454 \pm 0.059 \text{ GeV} \quad . \tag{11}$$

Further combination with the preliminary LEP average [11] of

$$M_W^{\text{LEP}} = 80.450 \pm 0.039 \text{ GeV} \tag{12}$$

assuming no correlated uncertainty gives

$$M_W = 80.451 \pm 0.032 \text{ GeV} \tag{13}$$

as the preliminary world average (with $\chi^2 = 0.003$). Figure 1 shows the W boson mass results, compared with the indirect value of 80.373 ± 0.023 GeV, derived from other electroweak measurements [11], as interpreted in the context of the standard model.

3 W Boson Width

The direct measurement of the W boson width is made by analysing W boson candidate events with transverse mass above the Jacobian peak near 80 GeV. The fitting range extends roughly between 100 GeV and 200 GeV, where the resolution effects from the Jacobian peak are small. The W boson width analysis shares most of the issues of W production and decay modelling and the detector response with the W boson mass analysis, and the sources of uncertainty are therefore similar.

As with the W boson mass analysis, the model parameters are constrained by analysis of internal data by each experiment separately. Therefore most of the uncertainties (those shown in Table 3) are uncorrelated. These uncertainties are also uncorrelated between the CDF electron and muon channel results.

The correlated sources of uncertainty are

- Parton distribution functions - The CDF and DØ analyses used different sets of PDFs to evaluate this uncertainty and quote different contributions. The W boson acceptance is similar in the direct measurements of the W boson width since both experiments require lepton $p_T > 20$ GeV and $|\eta| < 1$.

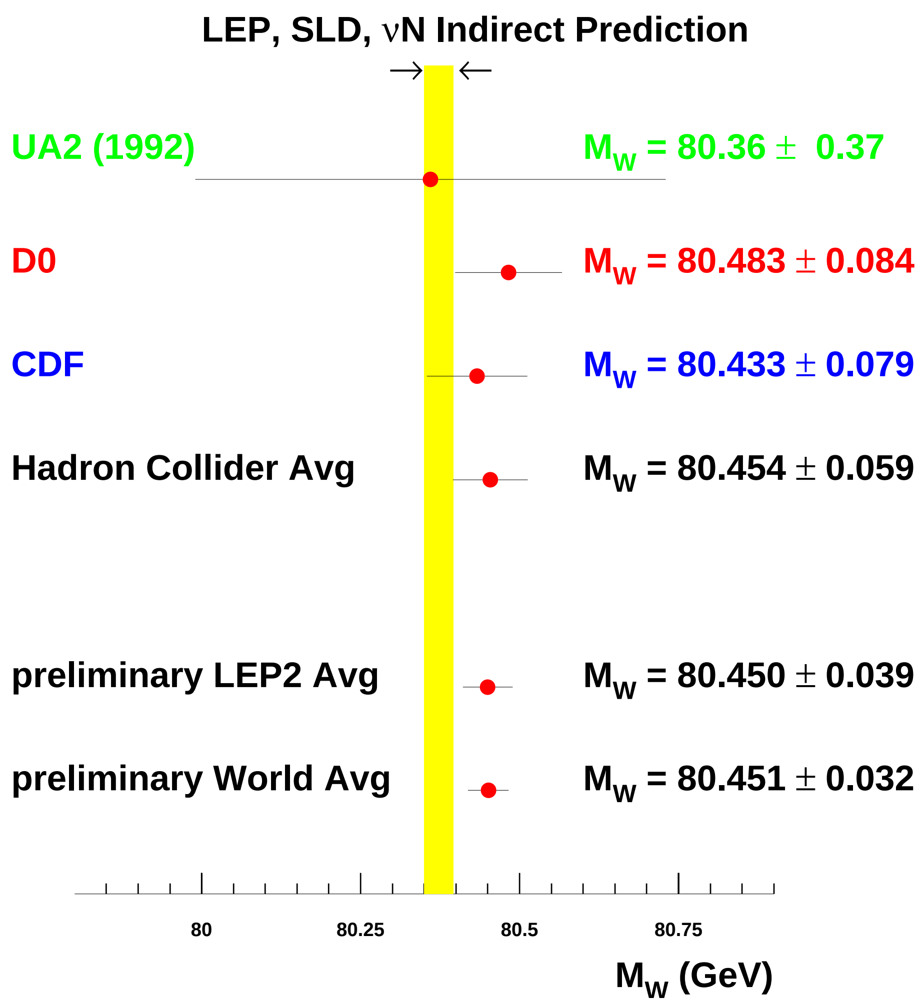

Figure 1: Direct measurements of the W boson mass compared with the standard model prediction based on other electroweak measurements.

Table 3: Uncorrelated uncertainties (MeV) in the CDF and DØ W boson width measurements. W boson decay channels used (e, μ) are listed separately.

Source	CDF μ	CDF e	DØ e
W statistics	195	125	142
Lepton energy scale	15	20	42
Lepton E or p_T non-linearity	5	60	-
Recoil model	90	60	59
$p_T(W)$	70	55	12
Backgrounds	50	30	42
Detector modelling, lepton ID	40	30	10
Lepton resolution	20	10	27
Parton luminosity slope	-	-	28

- W boson mass
- QED radiative corrections

The Run 1 direct W boson width measurements from CDF [9] and DØ [12] are

$$
\begin{aligned}
\Gamma_W &= 2.05 \pm 0.13 \text{ GeV (CDF)} \\
\Gamma_W &= 2.231^{+0.175}_{-0.170} \text{ GeV (DØ)}
\end{aligned}
\tag{14}
$$

where the total uncertainty is quoted. The correlated uncertainties for the two measurements are shown in Table 4. The likelihood fit returns a slightly asymmetric statistical error for the DØ result. We symmetrize it by taking

Table 4: Systematic uncertainties (MeV) from correlated sources in the W boson width measurements.

Source	CDF	DØ
PDF	15	27
Radiative Corrections	10	10
W boson mass	10	15

the arithmetic average and combine in quadrature with the total systematic uncertainty to obtain a total uncertainty of 173 MeV for the DØ result. We use the procedure described in Section 2.3 to construct the covariance matrix, and use it to obtain the combined result

$$\Gamma_W = 2.115 \pm 0.105 \ \ \text{GeV} \tag{15}$$

with $\chi^2 = 0.7$ and probability of 40%. The square root of the off-diagonal covariance matrix element gives the total correlated error of 26 MeV.

Combination with the preliminary LEP average [11] of

$$\Gamma_W^{\text{LEP}} = 2.150 \pm 0.091 \ \ \text{GeV} \tag{16}$$

assuming no correlated uncertainty gives

$$\Gamma_W = 2.135 \pm 0.069 \ \ \text{GeV} \tag{17}$$

as the preliminary world average (with $\chi^2 = 0.063$). Figure 2 shows the W boson width results, compared with the standard model prediction of 2.0927 ± 0.0025 GeV [13].

4 Joint Covariance Matrix of W Boson Mass and Width

In previous sections of this document we reported on the combination of the individual direct measurements of the W boson mass and width. In the W mass analysis, the observed W boson mass value, which is obtained by fitting the data with Monte Carlo templates, depends on the value of the W boson width assumed in the Monte Carlo. The W width is treated as an external input in the said analysis. In the same way, the W width analysis treats the W mass as an external input. These individual analyses do not provide the correlated uncertainty between the W mass and width measurements. The covariance matrix of these measurements is needed if they are to be used to perform a multivariate fit or consistency test with other data or theoretical predictions.

In this section we perform the error analysis for a joint direct measurement of the W boson mass and the width. In this error analysis, we do not

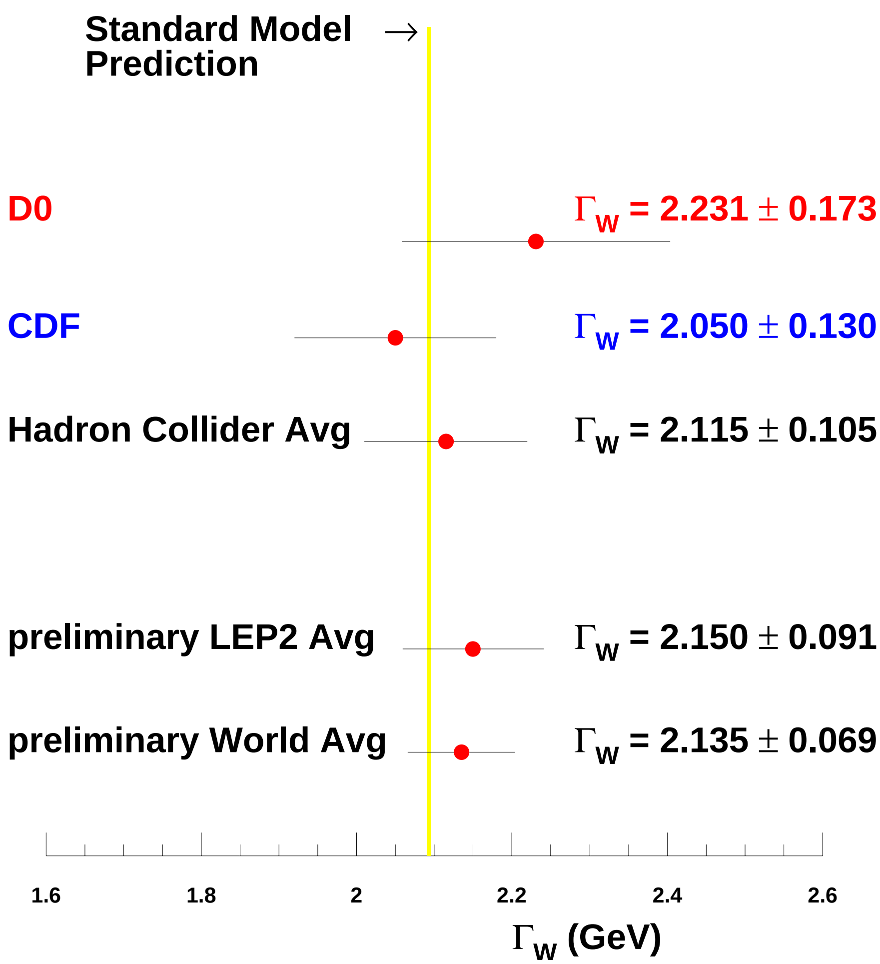

Figure 2: Direct measurements of the W boson width compared with the standard model prediction.

allow external constraints on the mass and width parameters: instead we propagate the uncertainties on the direct observables to the uncertainties on the extracted theory parameters. This procedure will give us the uncertainties on M_W and Γ_W extracted from "Tevatron data only", as well as their covariance.

To describe our procedure, we introduce the following terminology to distinguish between the observables called M_W and Γ_W (which are returned by the fits to the data spectra) and the theory parameters of the same names (which we want to extract). We define the vector of observables $\vec{o} = (M_W^o, \Gamma_W^o)$ and the vector of theory parameters $\vec{t} = (M_W^t, \Gamma_W^t)$. We approximate the functional dependence $\vec{o}\ (\ \vec{t}\ )$ by a linear dependence, so that $\vec{o}$ and $\vec{t}$ are related by a linear transformation. For the error analysis, we are interested in transforming the variations in $\vec{o}$ to variations in $\vec{t}$. This transformation is given by the matrix of derivatives $\Delta \equiv \partial\vec{o}/\partial\vec{t}$

$$\delta\vec{o} = \Delta\ \delta\vec{t} \tag{18}$$

The matrix of derivatives Δ is defined as

$$\Delta = \begin{pmatrix} \frac{\partial M_W^o}{\partial M_W^t} & \frac{\partial M_W^o}{\partial \Gamma_W^t} \\ & \\ \frac{\partial \Gamma_W^o}{\partial M_W^t} & \frac{\partial \Gamma_W^o}{\partial \Gamma_W^t} \end{pmatrix} \tag{19}$$

The values of the matrix elements of Δ have been derived by generating simulated data with different theory values of M_W and Γ_W, and fitting these simulated data to obtain the resulting observed values. The information from these Monte Carlo experiments has already been presented in previous sections. The fits to simulated data have demonstrated that the diagonal elements of Δ are unity. The off-diagonal element $\frac{\partial M_W^o}{\partial \Gamma_W^t}$ is given by the 10 MeV variation in observed M_W due to a 60 MeV variation in Γ_W^t [3], and the off-diagonal element $\frac{\partial \Gamma_W^o}{\partial M_W^t}$ is given by the mean variation of 13 MeV in observed Γ_W for a 39 MeV variation in M_W^t [9, 12]. Thus Δ is given by

$$\Delta = \begin{pmatrix} 1 & 0.17 \\ 0.33 & 1 \end{pmatrix} \tag{20}$$

We invert Eqn. 18 to obtain $\Delta^{-1}\ \delta\vec{o} = \delta\vec{t}$ and take the expection value of the product of each vector and its transpose

$$\Delta^{-1}\ <\delta\vec{o}\,(\delta\vec{o})^T>\ (\Delta^{-1})^T\ = <\delta\vec{t}\,(\delta\vec{t})^T> \tag{21}$$

where T denotes the transpose and $< ... >$ denotes the expection value. The LHS contains the covariance matrix of the observables $< \delta\vec{o}\ (\delta\vec{o})^T >$, and we identify the RHS with the covariance matrix of the extracted theory parameters.

The diagonal elements of $< \delta\vec{o}\ (\delta\vec{o})^T >$ are given by the variances of the individual Tevatron averages of the direct W boson mass and width (see Eqns. 9 and 15), excluding the error contribution to M_W due to Γ_W and vice-versa. In order to evaluate the off-diagonal matrix element, we analyse the various contributions to the respective variances. The observables are obtained from fits to disjoint data samples[1], so that their statistical uncertainties are uncorrelated. However, the observed values of M_W and Γ_W depend on the same detector parameters (such as energy scales and resolutions) and the same theoretical parameters (such as parton distribution functions and QED radiative corrections). Hence the uncertainties in these "nuisance" parameters propagate into correlated uncertainties between the observables.

To evaluate the off-diagonal term, we first find the uncertainty contribution to the observed M_W and Γ_W due to each of these nuisance parameters. The W mass and width analyses were performed in a closely related manner, using the same simulation programs, and so expect the uncertainty contributions due to the nuisance parameters to be completely correlated between the observed M_W and Γ_W. We evaluate each of these contributions to the CDF+DØ averages by (i) removing the respective contribution from the CDF and DØ results separately, (ii) recomputing the total error on the CDF+DØ average, and (iii) taking the difference in quadrature between the original total error and the reduced total error. Table 5 shows the uncertainty contributions from each correlated source to the CDF+DØ averages.

We use the information from Table 5 to evaluate the covariance term

$$< \delta M_W^o\ \delta\Gamma_W^o > = \sum_i\ \delta_i M_W^o\ \delta_i\Gamma_W^o \tag{22}$$

where the sum is performed over the various sources in Table 5, and $\delta_i M_W^o$ and $\delta_i\Gamma_W^o$ are the respective error contributions to M_W^o and Γ_W^o from source i. In this sum, the relative sign of each pair of factors $\delta_i M_W^o$ and $\delta_i\Gamma_W^o$

[1]The W mass fits are performed with the data satisfying $m_T < 90$ GeV or lepton $p_T < 50$ GeV, while the fits for the W width are performed with data satisfying $m_T > 100$ GeV.

Table 5: Correlated uncertainties (MeV) in the CDF+DØ averages due to nuisance parameters.

Source	M_W	Γ_W
Lepton scale	37	17
Lepton resolution	12	11
$p_T(W)$	9	24
Recoil model	20	35
Detector modelling, selection bias	6	13
QED radiative correction	11	10

determines the sign of the covariance contribution. $\delta_i M_W^o$ and $\delta_i \Gamma_W^o$ have the same sign in all cases. In the cases of the lepton energy scale, lepton energy resolution, $p_T(W)$ and recoil modelling, an increase in the respective parameter increases the observed values of both M_W and Γ_W. Similarly, in the cases of detector modelling, selection bias and QED radiative correction, the bias in the shape of the m_T or lepton p_T spectrum affects both observables in the same direction.

Table 6: Uncorrelated systematic uncertainties (MeV) in the CDF+DØ averages.

Source	M_W	Γ_W
Backgrounds	6	21
PDF, parton luminosity	12	22

Table 6 shows the systematic error contributions due to PDFs and backgrounds. We do not expect a strong correlation between the error contributions to the observed mass and width from these sources, because the observables are derived from different ranges in m_T. Thus, in the case of the PDFs, a different x range is relevant in each case. Furthermore, in the case of the W mass, the uncertainty in the PDFs propagates mainly through acceptance effects, while in the case of the W width, the main effect is through the relative normalization of the high and low m_T regions. In the case of backgrounds, QCD jet misidentification produces the dominant background whose shape is determined independently in the different m_T regions. The sensitivity to the background shape and normalization is different in the fits for the mass and the width, since the shapes of the signal distributions are very different in the respective fitting windows. On the basis of these arguments, we take the contributions in Table 6 to be uncorrelated. They are not used directly in this joint error analysis; we present them here for completeness and future reference.

Evaluating Eqn. 22, we find $< \delta M_W^o \, \delta \Gamma_W^o > = 43^2$ MeV^2, and the covariance matrix for M_W^o and Γ_W^o is

$$< \delta\vec{o}\,(\delta\vec{o})^T > = \begin{pmatrix} (59^2 - 10^2)\,\mathrm{MeV}^2 & 43^2\,\mathrm{MeV}^2 \\ 43^2\,\mathrm{MeV}^2 & (105^2 - 13^2)\,\mathrm{MeV}^2 \end{pmatrix} \tag{23}$$

Substituting this result and Δ into Eqn. 21 gives the covariance matrix for the extracted theory parameters M_W^t and Γ_W^t

$$< \delta\vec{t}\,(\delta\vec{t})^T > = \begin{pmatrix} (59\,\mathrm{MeV})^2 & -(33\,\mathrm{MeV})^2 \\ -(33\,\mathrm{MeV})^2 & (106\,\mathrm{MeV})^2 \end{pmatrix} \tag{24}$$

Since the variances of M_W^t and Γ_W^t in the joint error analysis are only slightly different compared to when these parameters are extracted separately, we do not recompute the central values.

The negative sign of the covariance between M_W^t and Γ_W^t can be understood as follows: a higher value of the theory mass parameter increases the predicted number of events at high m_T, causing the inferred Γ_W^t to reduce (given the number of observed events at high m_T). Similarly, a higher value of the theory width parameter increases the expected number of events on the high side of the Jacobian edge, causing the inferred M_W^t to reduce (given the observed position of the Jacobian edge).

Finally, it is of interest for future, higher precision measurements of M_W and Γ_W to pursue this joint analysis technique. We expect most error contributions to scale with the statistics of the data. Assumptions that are made in providing external input for Γ_W in the M_W analysis are not necessary in this joint analysis technique. We also note that there is almost no loss of precision compared to the individual measurements. While this may seem surprising, the reason is the positive covariance induced between M_W^o and Γ_W^o by the uncertainties in the nuisance parameters. This means that an error in any of the nuisance parameters moves M_W and Γ_W in the same direction. But since an increase in one causes the other to reduce as mentioned above, this overall negative feedback suppresses the systematic uncertainties from the nuisance parameters on both M_W^t and Γ_W^t. This reduction in other systematic errors compensates for the information lost in excluding external mass and width input.

5 Indirect W Boson Width

The CDF and DØ measurements of R (see Eqn. 4) have been presented [14, 15] and combined [16] elsewhere. We reproduce here some of the discussion from [16] and then combine the direct measurement of Γ_W with Γ_W extracted from R assuming the validity of the standard model.

The CDF and DØ measurement of R in the electron channel are

$$\begin{aligned} R &= 10.38 \pm 0.14\,(\text{stat}) \pm 0.17\,(\text{syst}) \quad (\text{CDF}) \\ R &= 10.49 \pm 0.14\,(\text{stat}) \pm 0.21\,(\text{syst}) \quad (\text{DØ}) \quad . \end{aligned} \tag{25}$$

The systematics due to the choice of PDF (0.3%), the uncertainty in M_W (0.1%) and higher-order electroweak corrections (1.0%) are treated as correlated uncertainties between the CDF and DØ measurements, to obtain a total correlated uncertainty of 1.0%. The average R value [16] is

$$\begin{aligned} R &= 10.42 \pm 0.15\,(\text{uncorrelated}) \pm 0.11\,(\text{correlated}) \\ &= 10.42 \pm 0.18 \quad . \end{aligned} \tag{26}$$

In the extraction of Γ_W from R, the $Z \to ee$ branching ratio is taken from the PDG [17] to be $(3.366 \pm 0.008)\%$. The calculated value of the W boson leptonic partial width $\Gamma(W \to e\nu) = 226.4 \pm 0.7$ MeV [18]. The inclusive cross section ratio σ_W/σ_Z is calculated at NNLO using Van Neerven *et al.* [19], with the inputs

- $M_W = 80.396 \pm 0.061$ GeV
- $M_Z = 91.187 \pm 0.007$ GeV
- $\Gamma_W = 2.06 \pm 0.05$ GeV
- $\Gamma_Z = 2.490 \pm 0.007$ GeV
- $\sin^2\theta_W = 0.23124 \pm 0.00024$

and the renormalization and factorization scales set to the boson mass. The calculated value of ratio of inclusive cross sections is found to be

$$\frac{\sigma_W}{\sigma_Z} = 3.362 \pm 0.015 \tag{27}$$

The dominant uncertainties (quoted in parentheses) in the calculation of the cross section ratio are due to PDFs (0.45%), M_W (0.09%), factorization scale (0.12%), renormalization scale (0.06%) and $\sin^2\theta_W$ (1.43%).

The value of the $W \to e\nu$ branching ratio extracted from the combined CDF and DØ measurement of R is

$$B(W \to e\nu) = (10.43 \pm 0.25)\ \% \tag{28}$$

The extracted value of Γ_W is

$$\Gamma_W = 2.171 \pm 0.052 \text{ GeV} \quad . \tag{29}$$

In order to combine this indirect measurement with the direct measurement of Γ_W, we assume that the uncertainties due to PDF variation are uncorrelated since the measured quantities are quite different. Electroweak corrections, factorization and renormalization scales and $\sin^2\theta_W$ also play no significant role in the direct Γ_W measurement. The uncertainties in R and σ_W/σ_Z due to M_W variation are of the same magnitude and sign[2] and therefore cancel in $B(W \to e\nu)$. The uncertainty in the standard model calculation of $\Gamma(W \to e\nu)$ due to uncertainty in M_W is 0.3% which is transferred to the extracted Γ_W as a 7 MeV uncertainty. This is anti-correlated [3] with the corresponding M_W uncertainty on the direct Γ_W measurement (13 MeV). Ignoring this anti-correlation, we find the Tevatron combined (direct and indirect) result

$$\Gamma_W = 2.160 \pm 0.047 \text{ GeV} \quad . \tag{30}$$

Further combining with the preliminary LEP direct measurement gives

$$\Gamma_W = 2.158 \pm 0.042 \text{ GeV} \quad . \tag{31}$$

[2]With increasing M_W, R reduces due to increased acceptance, and σ_W also reduces.

[3]With increasing M_W, the calculated $\Gamma(W \to e\nu)$ increases, whereas the directly measured Γ_W decreases.

6 Extraction of W Leptonic Width

We may use the extracted value of the $W \to e\nu$ branching ratio (Eqn. 28) and the directly measured total W width (Eqn. 15) to obtain a measurement of the W leptonic partial width

$$\begin{aligned} \Gamma(W \to e\nu) &= \Gamma_W \times B(W \to e\nu) \\ &= 220.6 \pm 12.2 \text{ MeV} \end{aligned} \tag{32}$$

where we have combined the fractional uncertainties in quadrature. As mentioned above, there is a small anti-correlation between the errors of the two input factors, which we have ignored. The fractional uncertainty in the direct Γ_W (5.0%) dominates over the fractional uncertainty in $B(W \to e\nu)$ (2.4%). This measurement of $\Gamma(W \to e\nu)$ is in good agreement with the standard model calculation given in Sec. 5.

7 Conclusion

We have presented the Run 1 results on the W boson mass and width from the CDF and DØ experiments, and examined their sources of uncertainty to identify the correlated components. We have used the covariance matrix technique to combine the respective measurements from the two experiments. The χ^2 probability for each combination is good indicating that the measurements are consistent. We have also reported the joint covariance matrix of the W mass and direct W width measurements. Finally, we have extracted the W leptonic partial width from the measured total W width and the leptonic branching ratio.

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